Unidad 4: Métodos Numéricos

Tema 4.2: Método de Runge Kutta

Ejemplo de solución de una ecuación Diferencial de 1er Orden, a partir de una condición inicial dada, por el Método de Runge - Kutta para encontrar una curva solución

$$y' = \frac{dy}{dx} = 2xy \quad ; \quad y(1) = 1$$

$$x_{n+1} = x_n + h$$

$$y_{n+1} = y_n + \frac{1}{6} \cdot (k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = h \cdot f(x_n, y_n)$$

$$k_2 = h \cdot f\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = h \cdot f\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = h \cdot f(x_n + h, y_n + k_3)$$

h= 0.05

n	Xn	Yn	k1	k2	k3	k4
0	1.00	1.00	0.1000	0.1076	0.1080	0.1163
1	1.05	1.11	0.1163	0.1254	0.1258	0.1357
2	1.10	1.23	0.1357	0.1464	0.1470	0.1588
3	1.15	1.38	0.1588	0.1715	0.1723	0.1863
4	1.20	1.55	0.1863	0.2016	0.2026	0.2194
5	1.25	1.76	0.2194	0.2378	0.2389	0.2592
6	1.30	1.99	0.2592	0.2813	0.2828	0.3073
7	1.35	2.28	0.3073	0.3341	0.3359	0.3657
8	1.40	2.61	0.3656	0.3982	0.4005	0.4368
9	1.45	3.01	0.4367	0.4764	0.4794	0.5237
10	1.50	3.49	0.5236	0.5722	0.5759	0.6303
11	1.55	4.07	0.6301	0.6899	0.6946	0.7616
12	1.60	4.76	0.7614	0.8352	0.8412	0.9240
13	1.65	5.60	0.9237	1.0151	1.0228	1.1256
14	1.70	6.62	1.1253	1.2389	1.2487	1.3769
15	1.75	7.87	1.3765	1.5183	1.5309	1.6914
16	1.80	9.39	1.6908	1.8686	1.8848	2.0864
17	1.85	11.27	2.0857	2.3094	2.3304	2.5848
18	1.90	13.60	2.5838	2.8665	2.8937	3.2161
19	1.95	16.49	3.2147	3.5734	3.6088	4.0189
20	2.00	20.09	4.0170	4.4740	4.5203	5.0441

Método de Runge – Kutta ejemplo resuelto con detalles

$y' = 2xy \quad ; \quad f(x,y) = 2xy \quad ; \quad y(1) = 1 \quad ; \quad h = 0.10$

$$k_1 = hf(x_n, y_n)$$
$$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$
$$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$
$$k_4 = hf(x_n + h, y_n + k_3)$$
$$k_{prom} = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

	$x_{n+1} = x_n + h$	$y_{n+1} = y_n + k_{prom}$	k_1	k_2	k_3	k_4	k_{prom}
0	$x_0 = 1.0000$	$y_0 = 1.0000$	$x_0 = 1.0000$ $y_0 = 1.0000$ $f = 2.0000$ $k_1 = 0.2000$	$x_0 + \frac{h}{2} = 1.0500$ $y_0 + \frac{k_1}{2} = 1.1000$ $f = 2.3100$ $k_2 = 0.2310$	$x_0 + \frac{h}{2} = 1.0500$ $y_0 + \frac{k_2}{2} = 1.1155$ $f = 2.34255$ $k_3 = 0.234255$	$x_0 + h = 1.1000$ $y_0 + k_3 = 1.2342$ $f = 2.715361$ $k_4 = 0.271536$	$k_{prom} = 0.233674$
1	$x_1 = 1.1000$	$y_1 = 1.2337$	$x_1 = 1.1000$ $y_1 = 1.2337$ $f = 2.714083$ $k_1 = 0.27141$	$x_1 + \frac{h}{2} = 1.1500$ $y_1 + \frac{k_1}{2} = 1.3694$ $f = 3.14962$ $k_2 = 0.314962$	$x_1 + \frac{h}{2} = 1.1500$ $y_1 + \frac{k_2}{2} = 1.3911$ $f = 3.199716$ $k_3 = 0.319971$	$x_1 + h = 1.2000$ $y_1 + k_3 = 1.5537$ $f = 3.7288$ $k_4 = 0.37288$	$k_{prom} = 0.319024$
2	$x_2 = 1.2000$	$y_2 = 1.5527$	$x_2 = 1.2000$ $y_2 = 1.5527$ $f = 3.72648$ $k_1 = 0.37265$	$x_2 + \frac{h}{2} = 1.2500$ $y_2 + \frac{k_1}{2} = 1.7390$ $f = 4.34756$ $k_2 = 0.434756$	$x_2 + \frac{h}{2} = 1.2500$ $y_2 + \frac{k_2}{2} = 1.7701$ $f = 4.42519$ $k_3 = 0.44252$	$x_2 + h = 1.3000$ $y_2 + k_3 = 1.99522$ $f = 5.187572$ $k_4 = 0.51876$	$k_{prom} = 0.440994$
3	$x_3 = 1.3000$	$y_3 = 1.9937$					

Ejemplos para la clase:

Use el método de Runge-Kutta con h=0.1 para obtener una aproximación, con cuatro decimales, al valor indicado en los siguientes problemas de valor inicial

$y' = 2x - 3y + 1$ $y(1) = 5$ $y(1.5) = ?$	$y' = 1 + y^2$ $y(0) = 0$ $y(0.5) = ?$	$y' = (x - y)^2$ $y(0) = 0.5$ $y(0.5) = ?$	$y' = xy^2 - \frac{y}{x}$ $y(1) = 1$ $y(1.5) = ?$

Respuestas

x_n	y_n	x_n	y_n	x_n	y_n	x_n	y_n
1.00	5.0000	0.00	0.0000	0.00	0.5000	1.00	1.0000
1.10	3.9724	0.10	0.1003	0.10	0.5213	1.10	1.0101
1.20	3.2284	0.20	0.2027	0.20	0.5358	1.20	1.0417
1.30	2.6945	0.30	0.3093	0.30	0.5443	1.30	1.0989
1.40	2.3163	0.40	0.4228	0.40	0.5482	1.40	1.1905
1.50	2.0533	0.50	0.5463	0.50	0.5493	1.50	1.3333