Tema 7.2 Ejemplo de Movimiento en el espacio.

Calcule las componentes tangencial y normal de la aceleración, la curvatura, y los vectores unitarios, T, N, y B para una partícula que se mueve por la curva dada en el punto indicado

$$\vec{r}(t) = \left\langle t\,,\, \frac{1}{2}t^2\,,\, \frac{1}{3}t^3 \right\rangle \quad ; \quad \left(1\,,\, \frac{1}{2}\,,\, \frac{1}{3}\right)$$

$$\vec{v}(t) = \left\langle 1\,,\, t\,,\, t^2 \right\rangle \rightarrow \vec{v}(1) = \langle 1\,,1\,,1\rangle$$

$$|\vec{v}(t)| = \sqrt{1 + t^2 + t^4} \rightarrow |\vec{v}(1)| = \sqrt{1+1+1} = \sqrt{3}$$

$$\vec{a}(t) = \langle 0\,,1\,,2t\rangle \rightarrow \vec{a}(1) = \langle 0\,,1\,,2\rangle$$

$$\vec{v}\circ\vec{a} = \langle 1\,,1\,,1\rangle \circ \langle 0\,,1\,,2\rangle = 0+1+2 = 3$$

$$a_T = \frac{\vec{v}\circ\vec{a}}{|\vec{v}|} = \frac{3}{\sqrt{3}} = \sqrt{3} \rightarrow \underline{\underline{a_T = \sqrt{3}}}$$

$$\vec{v}\times\vec{a} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 0 & 1 & 2 \end{vmatrix} = \langle 1,-2,1\rangle$$

$$|\vec{v}\times\vec{a}| = \sqrt{1+4+1} = \sqrt{6}$$

$$a_N = \frac{|\vec{v}\times\vec{a}|}{|\vec{v}|} = \frac{\sqrt{6}}{\sqrt{3}} = \sqrt{2} \rightarrow \underline{\underline{a_N = \sqrt{2}}}$$

$$k = \frac{|\vec{v}\times\vec{a}|}{|\vec{v}|^3} = \frac{\sqrt{6}}{\left(\sqrt{3}\right)^3} = \frac{\sqrt{3}\sqrt{2}}{\sqrt{3}\cdot 3} = \underline{\underline{\frac{\sqrt{2}}{3} = k}}$$

$$\vec{T} = \frac{\vec{r}\,'}{|\vec{r}\,'|} = \frac{\vec{v}}{|\vec{v}|} = \frac{\langle 1\,,1\,,1\rangle}{\sqrt{3}} = \underline{\underline{\left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle = \vec{T}}}$$

$$\vec{a} = a_x\vec{i}\,\acute{a}_y\vec{j} \quad ; \quad \vec{a} = a_T\hat{T} + a_N\hat{N}$$

$$\vec{N} = \frac{\vec{a} - a_T\hat{T}}{a_N} = \frac{\langle 0\,,1\,,2\rangle - \sqrt{3}\left\langle \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle}{\sqrt{2}}$$

$$\vec{N} = \frac{\langle 0\,,1\,,2\rangle - \langle 1\,,1\,,1\rangle}{\sqrt{2}} = \underline{\underline{\left\langle \frac{-1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right\rangle = \vec{N}}}$$

$$\vec{B} = \hat{T}\times\vec{N} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{vmatrix}$$

$$\underline{\underline{\vec{B} = \left\langle \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right\rangle}}$$

Otra manera de calcular el vector N se muestra en la página siguiente,

Otra manera de hacerlo con la calculadora TI-89 ó Voyage 200 se muestra a continuación:

[t,t²/2,t³/3]→r(t)

d(r(t),t)→v(t)

norm(v(t))

v(t)/norm(v(t))→g(t) , que sería T(t)=v(t)/Iv(t)I

d(g(t),t)→h(t) , que sería dT/dt

h(t)/norm(h(t))→k(t) , que sería N(t)

k(1) , que sería N(1)

crossP(g(t),k(t)) , que sería B(t)

B(1)

$$\vec{N} = \frac{\vec{T}'}{\left|\vec{T}'\right|}$$

$$\vec{T}(t) = \frac{\vec{r}\,'}{\left|\vec{r}\,'\right|} = \frac{\left\langle 1, t, t^2 \right\rangle}{\sqrt{1+t^2+t^4}}$$

$$\vec{T}(t) = \left\langle \frac{1}{\sqrt{1+t^2+t^4}}, \frac{t}{\sqrt{1+t^2+t^4}}, \frac{t^2}{\sqrt{1+t^2+t^4}} \right\rangle$$

$$\vec{T}'(t) = \left\langle \frac{0 - \frac{2t+4t^3}{2\sqrt{1+t^2+t^4}}}{1+t^2+t^4}, \frac{\sqrt{1+t^2+t^4} - \frac{t\left(2t+4t^3\right)}{2\sqrt{1+t^2+t^4}}}{1+t^2+t^4}, \frac{2t\left(1+t^2+t^4\right) - t^2\,\frac{2t+4t^3}{2\sqrt{1+t^2+t^4}}}{1+t^2+t^4} \right\rangle$$

$$\vec{T}'(t) = \left\langle \frac{-\left(t+2t^3\right)}{\left(1+t^2+t^4\right)^{3/2}}, \frac{2+2t^2+2t^4-2t^2-4t^4}{2\left(1+t^2+t^4\right)^{3/2}}, \frac{4t+4t^3+4t^5-2t^3-4t^5}{2\left(1+t^2+t^4\right)^{3/2}} \right\rangle$$

$$\vec{T}'(t) = \left\langle \frac{-\left(t+2t^3\right)}{\left(1+t^2+t^4\right)^{3/2}}, \frac{2-2t^4}{2\left(1+t^2+t^4\right)^{3/2}}, \frac{4t+2t^3}{2\left(1+t^2+t^4\right)^{3/2}} \right\rangle$$

$$\vec{T}'(t) = \left\langle \frac{-t\left(1+2t^2\right)}{\left(1+t^2+t^4\right)^{3/2}}, \frac{1-t^4}{\left(1+t^2+t^4\right)^{3/2}}, \frac{t\left(t^2+2\right)}{\left(1+t^2+t^4\right)^{3/2}} \right\rangle$$

$$\vec{T}'(1) = \left\langle \frac{-3}{(3)^{3/2}}, \frac{0}{(3)^{3/2}}, \frac{3}{(3)^{3/2}} \right\rangle = \left\langle \frac{-1}{\sqrt{3}}, \frac{0}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle \quad \rightarrow \quad \left|\vec{T}'\right| = \sqrt{\frac{1}{3}+0+\frac{1}{3}} = \sqrt{\frac{2}{3}}$$

$$\vec{N}(1) = \frac{\vec{T}'(1)}{\left|\vec{T}'\right|} = \frac{\left\langle \frac{-1}{\sqrt{3}}, \frac{0}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle}{\sqrt{\frac{2}{3}}} = \sqrt{\frac{3}{2}}\left\langle \frac{-1}{\sqrt{3}}, \frac{0}{\sqrt{3}}, \frac{1}{\sqrt{3}} \right\rangle = \underline{\underline{\left\langle \frac{-1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right\rangle = \vec{N}(1)}}$$