Unidad 2 : Método de Series para Ecuaciones Lineales

Tema 3.4: Método de Series para EDL No Homogéneas de 2o orden

Ejemplo:

$$y'' - xy' - y = 3x^2 - 6x + 2$$

$$y = \sum_{n=0}^{\infty} c_n x^n \; ; \quad y' = \sum_{n=1}^{\infty} n c_n x^{n-1} \; ; \quad y'' = \sum_{n=2}^{\infty} n(n-1) c_n x^{n-2}$$

$$\sum_{n=2}^{\infty} n(n-1) c_n x^{n-2} - \sum_{n=1}^{\infty} n c_n x^n - \sum_{n=0}^{\infty} c_n x^n = 3x^2 - 6x + 2$$

$$n - 2 = k \quad ; \quad n = k \quad ; \quad n = k$$

$$n = k + 2$$

$$\sum_{k=0}^{\infty} (k+2)(k+1) c_{k+2} x^k - \sum_{k=1}^{\infty} k c_k x^k - \sum_{k=0}^{\infty} c_k x^k = 3x^2 - 6x + 2$$

$$(2c_2 - c_0) + \sum_{k=1}^{\infty} \left[(k+2)(k+1) c_{k+2} - (k+1) c_k\right] x^k = 3x^2 - 6x + 2$$

$$\underbrace{(2c_2 - c_0) x^0}_{=2} + \underbrace{[6c_3 - 2c_1] x^1}_{=-6} + \underbrace{[12c_4 - 3c_2] x^2}_{=3} + \sum_{k=3}^{\infty} \underbrace{(k+1)\left[(k+2) c_{k+2} - c_k\right] x^k}_{=0}$$

$$= 3x^2 - 6x + 2$$

$$\textit{hasta aqui vale } 1/3$$

$$2c_2 - c_0 = 2 \; ; \quad 2(3c_3 - c_1) = -6 \; ; \quad 3(4c_4 - c_2) = 3$$

$$2c_2 = c_0 + 2 \; ; \quad 3c_3 - c_1 = -3 \; ; \quad 4c_4 - c_2 = 1 \; ; \quad (k+1)[(k+2)c_{k+2} - c_k] = 0$$

$$c_2 = \frac{1}{2}c_0 + 1 \; ; \quad c_3 = \frac{1}{3}c_1 - 1 \; ; \quad c_4 = \frac{1}{4}c_2 + \frac{1}{4} \; ; \quad \underline{\underline{c_{k+2} = \frac{c_k}{k+2} \; ; \quad k \geq 3}}$$

$$c_4 = \frac{1}{4}\left(\frac{1}{2}c_0 + 1\right) + \frac{1}{4} = \frac{1}{2 \cdot 4}c_0 + \frac{1}{8}$$

$$k = 3 \; ; \quad c_5 = \frac{c_3}{5} = \frac{1}{5}\left(\frac{1}{3}c_1 - 1\right) = \frac{1}{3 \cdot 5}c_1 - \frac{1}{5}$$

$$k = 4 \; ; \quad c_6 = \frac{c_4}{6} = \frac{1}{6}\left(\frac{1}{2 \cdot 4}c_0 + \frac{1}{8}\right) = \frac{1}{2 \cdot 4 \cdot 6}c_0 + \frac{1}{6 \cdot 8}$$

$$k = 5 \; ; \quad c_7 = \frac{c_5}{7} = \frac{1}{7}\left(\frac{1}{3 \cdot 5}c_1 - \frac{1}{5}\right) = \frac{1}{3 \cdot 5 \cdot 7}c_1 - \frac{1}{5 \cdot 7}$$

hasta aqui vale $2/3$

$$y = \sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 + c_5 x^5 + c_6 x^6 + c_7 x^7 + \ldots$$

$$y = c_0 + c_1 x + \left(\frac{1}{2}c_0 + 1\right)x^2 + \left(\frac{1}{3}c_1 - 1\right)x^3 + \left(\frac{1}{2 \cdot 4}c_0 + \frac{1}{8}\right)x^4 +$$

$$\left(\frac{1}{3 \cdot 5}c_1 - \frac{1}{5}\right)x^5 + \left(\frac{1}{2 \cdot 4 \cdot 6}c_0 + \frac{1}{6 \cdot 8}\right)x^6 + \left(\frac{1}{3 \cdot 5 \cdot 7}c_1 - \frac{1}{5 \cdot 7}\right)x^7 +$$

$$y = c_0\left[1 + \frac{1}{2}x^2 + \frac{1}{2 \cdot 4}x^4 + \frac{1}{2 \cdot 4 \cdot 6}x^6 + \ldots\right]$$

$$+ c_1\left[x + \frac{1}{3}x^3 + \frac{1}{3 \cdot 5}x^5 + \frac{1}{3 \cdot 5 \cdot 7}x^7 + \ldots\right]$$

$$+ \left[x^2 - x^3 + \frac{1}{8}x^4 - \frac{1}{5}x^5 + \frac{1}{6 \cdot 8}x^6 - \frac{1}{5 \cdot 7}x^7 + \ldots\right]$$

Comandos para resolver EDLNH por series en el paquete Maple:

$$y'' - xy' - y = 3x^2 - 6x + 2$$

$$Order := 10;$$

$$ode := diff\left(y(x), x, x\right) - x * diff\left(y(x), x\right) - y(x) = 3 * x\text{^}2 - 6 * x + 2;$$

$$ans := dsolve(\{ode\}, y(x), type = series);$$

<u>Para la próxima clase estudiar las secciones:</u>

6.1 Zill	8.1	Nagle	Método de Series de Potencias
7.1 Zill	7.2	Nagle	Transformada de Laplace

<u>Tarea para entregar la próxima clase:</u>

Tarea No. 20 : Método de Series para EDL No-Homogéneas de 2º Orden

Ma-841 : ECUACIONES DIFERENCIALES

Tarea No. 20 : Método de Series para EDL No Homogéneas de 2o Orden

Resuelva las siguientes ED usando el Método de Series de Potencias

1) $y'' + xy' + x^2 y = 2x^2$

2) $y'' - 2xy = x^2$

3) $y'' + x^2 y = x^2 + x + 1$

4) $xy'' + x^2 y' = x^3 + 4x$

Respuestas:

1) $$y = c_0\left[1 - \frac{x^4}{12} + \frac{x^6}{90} + \frac{x^8}{3360} + \ldots\right] + c_1\left[x - \frac{x^3}{6} - \frac{x^5}{40} + \frac{x^7}{144} + \ldots\right] + \left[\frac{x^4}{6} - \frac{x^6}{45} - \frac{x^8}{1680} + \ldots\right]$$

2) $$y = c_0\left[1 + \frac{x^3}{3} + \frac{x^6}{45} + \frac{x^9}{1620} + \ldots\right] + c_1\left[x + \frac{x^4}{6} + \frac{x^7}{126} + \frac{x^{10}}{5670} + \ldots\right] + \left[\frac{x^4}{12} + \frac{x^7}{252} + \frac{x^{10}}{11340} + \ldots\right]$$

3) $$y = c_0\left[1 - \frac{x^4}{12} + \frac{x^8}{672} + \ldots\right] + c_1\left[x - \frac{x^5}{20} + \frac{x^9}{1440} + \ldots\right] + \left[\frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{12} - \frac{x^6}{60} - \frac{x^7}{252} - \frac{x^8}{672} + \frac{x^{10}}{5400} + \frac{x^{11}}{27720} \ldots\right]$$

4) $$y = c_0 + c_1\left[x - \frac{x^3}{6} + \frac{x^5}{40} - \frac{x^7}{336} + \frac{x^9}{3456} \ldots\right] + \left[2x^2 - \frac{x^4}{4} + \frac{x^6}{30} - \frac{x^8}{280} + \ldots\right]$$