# Unidad 5: Métodos Numéricos

## Tema 5.1 : Método de Euler

Recordando el desarrollo de una función en una serie de potencias de Taylor, tenemos que:

$$f(x) = c_0 + c_1(x - x_0) + c_2(x - x_0)^2 + c_3(x - x_0)^3 + c_4(x - x_0)^4 + \dots \quad ; \quad c_n = \frac{1}{n!}\frac{d^n f(x_0)}{dx^n}$$

$$f(x) = f(x_0) + (x - x_0)f'(x_0) + (x - x_0)^2 \frac{f''(x_0)}{2!} + (x - x_0)^3 \frac{f'''(x_0)}{3!} + \dots$$

y cambiándole de nombre a la función f(x) por y(x) tenemos que:

$$y(x) = y(x_0) + (x - x_0)y'(x_0) + (x - x_0)^2 \frac{y''(x_0)}{2!} + (x - x_0)^3 \frac{y'''(x_0)}{3!} + \dots$$

tomando la aproximación de 1er orden tenemos que:

$$y(x) = y(x_0) + (x - x_0)y'(x_0)$$

Consideremos ahora la solución del problema de valor inicial de 1er orden:

$$\frac{dy}{dx} = f(x, y) \quad ; \quad y(x_0) = x_0$$

combinando esta ecuación con la aproximación de Taylor de 1er orden tenemos:

$$\begin{aligned} y(x_1) &= y(x_0) + (x_1 - x_0)f(x_0, y_0) \\ y(x_2) &= y(x_1) + (x_2 - x_1)f(x_1, y_1) \\ y(x_3) &= y(x_2) + (x_3 - x_2)f(x_2, y_2) \end{aligned}$$

y simplificando la nomenclatura obtenemos que:

$$\begin{aligned} y_1 &= y_0 + hf(x_0, y_0) \quad ; \quad x_1 = x_0 + h \\ y_2 &= y_1 + hf(x_1, y_1) \quad ; \quad x_2 = x_1 + h \\ y_3 &= y_2 + hf(x_2, y_2) \quad ; \quad x_3 = x_2 + h \\ y_{n+1} &= y_n + hf(x_n, y_n) \quad ; \quad x_{n+1} = x_n + h \end{aligned}$$

| *Ejemplo del Método de Euler* | | | |
|---|---|---|---|
| | $y' = \frac{dy}{dx} = 2xy$ | $y(1.00) = 1.00$ | $h = 0.10$ |
| $n$ | $x_{n+1} = x_n + h$ | $y_{n+1} = y_n + hf(x_n, y_n)$ | $y' = f(x_n, y_n) = 2x_n y_n$ |
| 0 | $x_0 = \underline{\underline{1.00}}$ | $y_0 = \underline{\underline{1.0000}}$ | $y' = f(x_0, y_0) = 2x_0 y_0$<br>$f(1.00\ , 1.0000) = 2(1.00)(1.0000) = \underline{\underline{2.0000}}$ |
| 1 | $x_1 = x_0 + h$<br>$1.00 + 0.10 = \underline{\underline{1.10}}$ | $y_1 = y_0 + hf(x_0, y_0)$<br>$1.0000 + 0.1(2.0000) = \underline{\underline{1.2000}}$ | $y' = f(x_1, y_1) = 2x_1 y_1$<br>$f(1.10\ , 1.2000) = 2(1.10)(1.2000) = \underline{\underline{2.6400}}$ |
| 2 | $x_2 = x_1 + h$<br>$1.10 + 0.10 = \underline{\underline{1.20}}$ | $y_2 = y_1 + hf(x_1, y_1)$<br>$1.2000 + 0.1(2.6400) = \underline{\underline{1.4640}}$ | $y' = f(x_2, y_2) = 2x_2 y_2$<br>$f(1.20\ , 1.4640) = 2(1.20)(1.4640) = \underline{\underline{3.5136}}$ |
| 3 | $x_3 = x_2 + h$<br>$1.20 + 0.10 = \underline{\underline{1.30}}$ | $y_3 = y_2 + hf(x_2, y_2)$<br>$1.464 + 0.1(3.5136) = \underline{\underline{1.8154}}$ | $y' = f(x_3, y_3) = 2x_3 y_3$<br>$f(1.30\ , 1.8154) = 2(1.30)(1.8154) = \underline{\underline{4.7199}}$ |
| 4 | $x_4 = x_3 + h$<br>$1.30 + 0.10 = \underline{\underline{1.40}}$ | $y_4 = y_3 + hf(x_3, y_3)$<br>$1.8154 + 0.1(4.7199) = \underline{\underline{2.2874}}$ | $y' = f(x_4, y_4) = 2x_4 y_4$<br>$f(1.40\ , 2.2874) = 2(1.40)(2.2874) = \underline{\underline{6.4046}}$ |
| 5 | $x_5 = x_4 + h$<br>$1.40 + 0.10 = \underline{\underline{1.50}}$ | $y_5 = y_4 + hf(x_4, y_4)$<br>$2.2874 + 0.1(6.4046) = \underline{\underline{2.9278}}$ | $y' = f(x_5, y_5) = 2x_5 y_5$<br>$f(1.50\ , 2.9278) = 2(1.50)(2.9278) = \underline{\underline{8.7834}}$ |

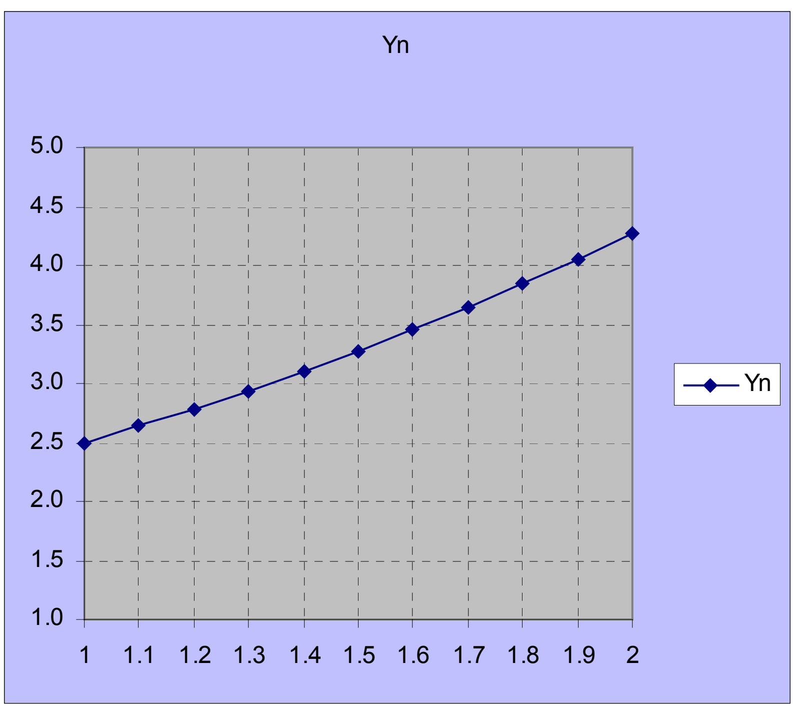
Ejemplo de solución de una ecuación Diferencial de 1er Orden, a partir de una condición inicial dada, por el Método de Euler para encontrar una curva solución

$$y' = \frac{dy}{dx} = x^{\frac{1}{2}} y^{\frac{1}{3}} \quad ; \quad y(1) = 2.5$$

$$x_{n+1} = x_n + h$$
$$y_{n+1} = y_n + h \cdot f(x_n, y_n)$$

| n | Xn | Yn | f(Xn,Yn) |
|---|---|---|---|
| 0 | 1 | 2.5000 | 1.3572 |
| 1 | 1.1 | 2.6357 | 1.4488 |
| 2 | 1.2 | 2.7806 | 1.5404 |
| 3 | 1.3 | 2.9346 | 1.6324 |
| 4 | 1.4 | 3.0979 | 1.7249 |
| 5 | 1.5 | 3.2704 | 1.8179 |
| 6 | 1.6 | 3.4522 | 1.9117 |
| 7 | 1.7 | 3.6433 | 2.0063 |
| 8 | 1.8 | 3.8440 | 2.1017 |
| 9 | 1.9 | 4.0541 | 2.1979 |
| 10 | 2 | 4.2739 | 2.2950 |

| | |
|---|---|
| Xo | 1 |
| Yo | 2.5 |
| h | 0.1 |
| f(Xn,Yn) | |



**Ejemplos para la clase:**

Use el método de Euler con h=0.1 para obtener una aproximación, con cuatro decimales, al valor indicado en los siguientes problemas de valor inicial

| | | | |
|---|---|---|---|
| $y' = 2x - 3y + 1$<br>$y(1) = 5$<br>$y(1.5) = ?$ | $y' = 1 + y^2$<br>$y(0) = 0$<br>$y(0.5) = ?$ | $y' = (x - y)^2$<br>$y(0) = 0.5$<br>$y(0.5) = ?$ | $y' = xy^2 - \frac{y}{x}$<br>$y(1) = 1$<br>$y(1.5) = ?$ |

Respuestas

| $x_n$ | $y_n$ | $x_n$ | $y_n$ | $x_n$ | $y_n$ | $x_n$ | $y_n$ |
|---|---|---|---|---|---|---|---|
| 1.00 | 5.0000 | 0.00 | 0.0000 | 0.00 | 0.5000 | 1.00 | 1.0000 |
| 1.10 | 3.9900 | 0.10 | 0.1005 | 0.10 | 0.5215 | 1.10 | 1.0095 |
| 1.20 | 3.2545 | 0.20 | 0.2030 | 0.20 | 0.5362 | 1.20 | 1.0404 |
| 1.30 | 2.7236 | 0.30 | 0.3098 | 0.30 | 0.5449 | 1.30 | 1.0967 |
| 1.40 | 2.3451 | 0.40 | 0.4234 | 0.40 | 0.5490 | 1.40 | 1.1866 |
| 1.50 | 2.0801 | 0.50 | 0.5470 | 0.50 | 0.5503 | 1.50 | 1.3260 |