

## Unidad 5: Métodos Numéricos

### Tema 5.2 : Método de Runge Kutta

Ejemplo de solución de una ecuación Diferencial de 1er Orden, a partir de una condición inicial dada, por el Método de Runge - Kutta para encontrar una curva solución

$$y' = \frac{dy}{dx} = 2xy \quad ; \quad y(1) = 1$$

$$x_{n+1} = x_n + h$$

$$y_{n+1} = y_n + \frac{1}{6} \cdot (k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = h \cdot f(x_n, y_n)$$

$$k_2 = h \cdot f\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = h \cdot f\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = h \cdot f(x_n + h, y_n + k_3)$$

h= 0.05

n	Xn	Yn	k1	k2	k3	k4
0	1.00	1.00	0.1000	0.1076	0.1080	0.1163
1	1.05	1.11	0.1163	0.1254	0.1258	0.1357
2	1.10	1.23	0.1357	0.1464	0.1470	0.1588
3	1.15	1.38	0.1588	0.1715	0.1723	0.1863
4	1.20	1.55	0.1863	0.2016	0.2026	0.2194
5	1.25	1.76	0.2194	0.2378	0.2389	0.2592
6	1.30	1.99	0.2592	0.2813	0.2828	0.3073
7	1.35	2.28	0.3073	0.3341	0.3359	0.3657
8	1.40	2.61	0.3656	0.3982	0.4005	0.4368
9	1.45	3.01	0.4367	0.4764	0.4794	0.5237
10	1.50	3.49	0.5236	0.5722	0.5759	0.6303
11	1.55	4.07	0.6301	0.6899	0.6946	0.7616
12	1.60	4.76	0.7614	0.8352	0.8412	0.9240
13	1.65	5.60	0.9237	1.0151	1.0228	1.1256
14	1.70	6.62	1.1253	1.2389	1.2487	1.3769
15	1.75	7.87	1.3765	1.5183	1.5309	1.6914
16	1.80	9.39	1.6908	1.8686	1.8848	2.0864
17	1.85	11.27	2.0857	2.3094	2.3304	2.5848
18	1.90	13.60	2.5838	2.8665	2.8937	3.2161
19	1.95	16.49	3.2147	3.5734	3.6088	4.0189
20	2.00	20.09	4.0170	4.4740	4.5203	5.0441

<b>Método de Runge – Kutta</b> <b>ejemplo resuelto con detalles</b> $y' = 2xy$ ; $f(x, y) = 2xy$ ; $y(1) = 1$ ; $h = 0.10$			$k_1 = hf(x_n, y_n)$ $k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$		$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$ $k_4 = hf(x_n + h, y_n + k_3)$		
			$k_{prom} = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$				
	$x_{n+1} = x_n + h$	$y_{n+1} = y_n + k_{prom}$	$k_1$	$k_2$	$k_3$	$k_4$	$k_{prom}$
0	$x_0 = 1.0000$	$y_0 = 1.0000$	$x_0 = 1.0000$ $y_0 = 1.0000$ $f = 2.0000$ $k_1 = 0.2000$	$x_0 + \frac{h}{2} = 1.0500$ $y_0 + \frac{k_1}{2} = 1.1000$ $f = 2.3100$ $k_2 = 0.2310$	$x_0 + \frac{h}{2} = 1.0500$ $y_0 + \frac{k_2}{2} = 1.1155$ $f = 2.34255$ $k_3 = 0.234255$	$x_0 + h = 1.1000$ $y_0 + k_3 = 1.2342$ $f = 2.715361$ $k_4 = 0.271536$	$k_{prom} = 0.233674$
1	$x_1 = 1.1000$	$y_1 = 1.2337$	$x_1 = 1.1000$ $y_1 = 1.2337$ $f = 2.714083$ $k_1 = 0.27141$	$x_1 + \frac{h}{2} = 1.1500$ $y_1 + \frac{k_1}{2} = 1.3694$ $f = 3.14962$ $k_2 = 0.314962$	$x_1 + \frac{h}{2} = 1.1500$ $y_1 + \frac{k_2}{2} = 1.3911$ $f = 3.199716$ $k_3 = 0.319971$	$x_1 + h = 1.2000$ $y_1 + k_3 = 1.5537$ $f = 3.7288$ $k_4 = 0.37288$	$k_{prom} = 0.319024$
2	$x_2 = 1.2000$	$y_2 = 1.5527$	$x_2 = 1.2000$ $y_2 = 1.5527$ $f = 3.72648$ $k_1 = 0.37265$	$x_2 + \frac{h}{2} = 1.2500$ $y_2 + \frac{k_1}{2} = 1.7390$ $f = 4.34756$ $k_2 = 0.434756$	$x_2 + \frac{h}{2} = 1.2500$ $y_2 + \frac{k_2}{2} = 1.7701$ $f = 4.42519$ $k_3 = 0.44252$	$x_2 + h = 1.3000$ $y_2 + k_3 = 1.99522$ $f = 5.187572$ $k_4 = 0.51876$	$k_{prom} = 0.440994$
3	$x_3 = 1.3000$	$y_3 = 1.9937$					

**Ejemplos para la clase:**

Use el método de Runge-Kutta con  $h=0.1$  para obtener una aproximación, con cuatro decimales, al valor indicado en los siguientes problemas de valor inicial

$y' = 2x - 3y + 1$ $y(1) = 5$ $y(1.5) = ?$	$y' = 1 + y^2$ $y(0) = 0$ $y(0.5) = ?$	$y' = (x - y)^2$ $y(0) = 0.5$ $y(0.5) = ?$	$y' = xy^2 - \frac{y}{x}$ $y(1) = 1$ $y(1.5) = ?$																																																								
<b>Respuestas</b>																																																											
<table border="1"> <thead> <tr> <th><math>x_n</math></th> <th><math>y_n</math></th> </tr> </thead> <tbody> <tr><td>1.00</td><td>5.0000</td></tr> <tr><td>1.10</td><td>3.9724</td></tr> <tr><td>1.20</td><td>3.2284</td></tr> <tr><td>1.30</td><td>2.6945</td></tr> <tr><td>1.40</td><td>2.3163</td></tr> <tr><td>1.50</td><td>2.0533</td></tr> </tbody> </table>	$x_n$	$y_n$	1.00	5.0000	1.10	3.9724	1.20	3.2284	1.30	2.6945	1.40	2.3163	1.50	2.0533	<table border="1"> <thead> <tr> <th><math>x_n</math></th> <th><math>y_n</math></th> </tr> </thead> <tbody> <tr><td>0.00</td><td>0.0000</td></tr> <tr><td>0.10</td><td>0.1003</td></tr> <tr><td>0.20</td><td>0.2027</td></tr> <tr><td>0.30</td><td>0.3093</td></tr> <tr><td>0.40</td><td>0.4228</td></tr> <tr><td>0.50</td><td>0.5463</td></tr> </tbody> </table>	$x_n$	$y_n$	0.00	0.0000	0.10	0.1003	0.20	0.2027	0.30	0.3093	0.40	0.4228	0.50	0.5463	<table border="1"> <thead> <tr> <th><math>x_n</math></th> <th><math>y_n</math></th> </tr> </thead> <tbody> <tr><td>0.00</td><td>0.5000</td></tr> <tr><td>0.10</td><td>0.5213</td></tr> <tr><td>0.20</td><td>0.5358</td></tr> <tr><td>0.30</td><td>0.5443</td></tr> <tr><td>0.40</td><td>0.5482</td></tr> <tr><td>0.50</td><td>0.5493</td></tr> </tbody> </table>	$x_n$	$y_n$	0.00	0.5000	0.10	0.5213	0.20	0.5358	0.30	0.5443	0.40	0.5482	0.50	0.5493	<table border="1"> <thead> <tr> <th><math>x_n</math></th> <th><math>y_n</math></th> </tr> </thead> <tbody> <tr><td>1.00</td><td>1.0000</td></tr> <tr><td>1.10</td><td>1.0101</td></tr> <tr><td>1.20</td><td>1.0417</td></tr> <tr><td>1.30</td><td>1.0989</td></tr> <tr><td>1.40</td><td>1.1905</td></tr> <tr><td>1.50</td><td>1.3333</td></tr> </tbody> </table>	$x_n$	$y_n$	1.00	1.0000	1.10	1.0101	1.20	1.0417	1.30	1.0989	1.40	1.1905	1.50	1.3333
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