Ejemplo Resuelto del Método de Sustitución para Ecuaciones de Coeficientes Homogéneos

$$(4x-5y)dx+(5x-y)dy=0$$
$$(5x-y)dy=-(4x-5y)dx$$
$$\frac{dy}{dx}=\frac{5y-4x}{5x-y} \quad ;$$
$$aplicamos\_el\_cambio\_v=\frac{y}{x}$$
$$y=vx \quad ; \quad \frac{dy}{dx}=v+x\frac{dv}{dx}$$
$$sustituyendo\_en\_la\_ED:$$
$$v+x\frac{dv}{dx}=\frac{5vx-4x}{5x-vx}$$
$$v+x\frac{dv}{dx}=\frac{x(5v-4)}{x(5-v)}$$
$$v+x\frac{dv}{dx}=\frac{5v-4}{5-v}$$
$$x\frac{dv}{dx}=\frac{5v-4}{5-v}-v$$
$$x\frac{dv}{dx}=\frac{5v-4-5v+v^2}{5-v}$$
$$x\frac{dv}{dx}=\frac{v^2-4}{5-v}$$
$$\frac{5-v}{v^2-4}dv=\frac{dx}{x}$$
$$\frac{5-v}{(v+2)(v-2)}dv=\frac{dx}{x}$$
$$\int\frac{5-v}{(v+2)(v-2)}dv=\int\frac{dx}{x}$$

$$\frac{5-v}{(v+2)(v-2)}=\frac{A}{v+2}+\frac{B}{v-2}$$
$$descomponemos\_en\_fracciones$$
$$5-v=A(v-2)+B(v+2)$$
$$5-v=v(A+B)+(-2A+2B)$$
$$A+B=-1 \quad ; \quad -2A+2B=5$$
$$A=\frac{-7}{4} \quad ; \quad B=\frac{3}{4}$$
$$\frac{5-v}{(v+2)(v-2)}=\frac{-7}{4}\cdot\frac{1}{v+2}+\frac{3}{4}\cdot\frac{1}{v-2}$$
$$\int\frac{5-v}{(v+2)(v-2)}dv=\frac{-7}{4}\cdot\int\frac{1}{v+2}dv+\frac{3}{4}\cdot\int\frac{1}{v-2}dv$$
$$\frac{-7}{4}\cdot\int\frac{1}{v+2}dv+\frac{3}{4}\cdot\int\frac{1}{v-2}dv=\int\frac{dx}{x}$$
$$\frac{-7}{4}\ln(v+2)+\frac{3}{4}\ln(v-2)=\ln(x)+c_0$$
$$-7\ln\left(\frac{y}{x}+2\right)+3\ln\left(\frac{y}{x}-2\right)=4\ln(x)+4c_0$$
$$-7\ln\left(\frac{y+2x}{x}\right)+3\ln\left(\frac{y-2x}{x}\right)=4\ln(x)+c_1$$
$$-7\ln(y+2x)+3\ln(y-2x)=c_1$$
$$\ln\left(\frac{(y-2x)^3}{(y+2x)^7}\right)=c_1$$
$$\frac{(y-2x)^3}{(y+2x)^7}=e^{c_1}$$
$$(y-2x)^3=c_2(y+2x)^7$$